UDC 531.55

MOTION OF A SOLID BODY WITH A CAVITY PARTLY FILLED WITH A VISCOUS FLUID UNDER CONDITIONS OF TOTAL WEIGHTLESSNESS

PMM Vol. 43, No. 2, 1979, pp. 267-273

NGO ZUI KAN
(Voronezh)

(Received April 7, 1978)

The problem of joint motion of a solid body and viscous incompressible fluid which partly fills a cavity in the body under conditions of total weightlessness is considered.

This problem was analyzed in [1 —3] without taking into account surface tension under conditions of normal gravity. Forced rotation of a viscous fluid with surface tension was considered in [4, 5].

1. Statement of the problem. Let a solid body with a cavity partly filled with a viscous incompressible fluid rotate around a fixed point O under conditions of total weightlessness.

The slow motion of fluid in the cavity is defined by the Navier—Stokes equations which in the system of coordinates $Ox_1x_2x_3$ fixed to the body is of the form

$$\boldsymbol{\varepsilon}\times\mathbf{r}+\frac{\partial\mathbf{u}}{\partial t}=-\frac{1}{\rho}\nabla p+\nu\Delta\mathbf{u} \tag{1.1}$$

$$\operatorname{div}\mathbf{u}=0 \quad \text{in} \quad \Omega$$

where $\mathbf{u}$ is the vector of fluid particle relative velocity, $\boldsymbol{\varepsilon}$ is the body angular acceleration vector, $\mathbf{r}$ is the radius vector of fluid particles relative to point O, p is the pressure in the fluid, ρ is the fluid density, ν is the kinematic viscosity coefficient, and Ω is the region occupied by the fluid in equilibrium.

The boundary condition along the wetted part of the cavity wall S is of the form

$$\mathbf{u}=0 \quad \text{on} \quad S \tag{1.2}$$

In the curvilinear system of coordinates (ξ_1, ξ_2, ξ_3), such that point $(\xi_1, \xi_2, 0)$ lies on Γ_0 and coordinate ξ_3 is read along the outer normal n to Γ_0, with the Lamé coefficient $h_3 = 1$, the boundary condition at the free surface of the fluid is of the form [4]

$$\mathbf{u}_{1,3}+\mathbf{u}_{3,1}=\mathbf{u}_{2,3}+\mathbf{u}_{3,2}=0, \quad \int_{\Gamma_0}\mathbf{u}_3\,d\Gamma=0 \tag{1.3}$$

$$p-2\rho\nu\frac{\partial\mathbf{u}_3}{\partial\xi_3}=\sigma B_1N, \quad \frac{\partial N}{\partial t}=u_3 \quad \text{on} \quad \Gamma_0$$

where σ is the surface tension coefficient, N is the deviation of the fluid free surface in motion from that in equilibrium Γ_0, and B_1 is the differential operator of the elliptic type

$$B_1N=aN-\Delta_\Gamma N-\frac{1}{|\Gamma_0|}\int_{\Gamma_0}(aN-\Delta_\Gamma N)\,d\Gamma$$

$$a = -(k_1^2 + k_2^2 + \sigma^{-1}\partial p_0 / \partial n)$$

where k_1 and k_2 are the principal curvatures of surface Γ_0 of the fluid in equilibrium, and Δ_Γ is the Laplace — Beltrami operator.

It is assumed, as in [4, 5], that the free surface Γ_0 has no common points with the cavity wetted surface S, and that the fluid state of equilibrium is stable, i. e. that operator B_1 is positive definite.

Under conditions of total weightlessness $(g = 0)$ the equation of motion of the body with fluid is of the form [2, 3]

$$\mathbf{J}\cdot\boldsymbol{\varepsilon} + \rho \int_\Omega \left[\mathbf{r} \times \frac{\partial \mathbf{u}}{\partial t}\right] d\Omega = 0 \tag{1.4}$$

where $\mathbf{J}$ is the tensor of the moment of inertia of the system "body + fluid" relative to point O, and g is the acceleration of gravity.

Eliminating $\boldsymbol{\varepsilon}$ in Eq. (1.1) using Eq. (1.4), we obtain

$$\mathbf{r} \times \rho\mathbf{J}^{-1} \int_\Omega \left[\mathbf{r} \times \frac{\partial \mathbf{u}}{\partial t}\right] d\Omega + \frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho}\nabla p + \nu\Delta\mathbf{u} \tag{1.5}$$

We shall investigate problem (1.1) —(1.3), (1.5) of determination of the motion of fluid in the joint motion of body and fluid with initial conditions

$$\mathbf{u}(0) = \mathbf{u}_0, \quad N(0) = N_0 \tag{1.6}$$

Acceleration of the body is determined by the known velocity of fluid using formula (1.4).

2. Reduction of the problem to operator equations. For analyzing the problem equations we introduce the functional spaces considered in [6]. We denote by $W_2^{1,0}(\Omega)$ the closure in the norm of the Sobolev space $W_2^1(\Omega)$ of the totality of solenoidal vector functions $\mathbf{v}$ from $W_2^1(\Omega)$ which vanish in the neighborhood of surface S. By $L_2^\circ(\Omega)$ we understand the completion of $W_2^{1,0}(\Omega)$ by the norm of space $L_2(\Omega)$. The orthogonal complement to $L_2^\circ(\Omega)$ in $L_2^\circ(\Omega)$ is the closure of vector functions that are potential in Ω and equal zero on Γ_0 (see, e.g., [5]). Note that for vector functions from $L_2^\circ(\Omega)$ the normal component on S is zero.

Using the method set forth in [5, 6] we reduce the second of Eqs. (1.1) and Eq. (1.5) with boundary conditions (1.2) and (1.3) to two operator equations in $L_2(\Omega)$ and $W_2^{-1/2}(\Gamma_0)$ of the form

$$\begin{gathered}(I + B)\, d\mathbf{u}/dt + \nu A\mathbf{v} = 0 \\ \nu\, d\boldsymbol{\varphi}/dt + \sigma B_1 \Gamma(\mathbf{v} + T\boldsymbol{\varphi}) = 0 \\ \mathbf{u} = \mathbf{v} + T\boldsymbol{\varphi}, \quad B\mathbf{v} = \Pi\left\{\mathbf{r} \times \rho\mathbf{J}^{-1}\int_\Omega [\mathbf{r}_1 \times \mathbf{v}]\, d\Omega\right\}\end{gathered} \tag{2.1}$$

where B is the carry operator [7], Π is the operator of orthogonal projection of space $L_2(\Omega)$ on subspace $L_2^0(\Omega)$, Γ is the operator of the vector function trace on the free surface Γ_0, and A and T are operators generated by the auxilliary boundary value problems described in [5, 6]. Operator A is self-conjugate and positive definite in $L_2^\circ(\Omega)$, and has a completely continuous inverse operator of class σ_p $(\forall_p > 3/2)$. The linear operator T continuously acts from $W_2^{-1/2}(\Gamma_0)$ in $W_2^{1,0}$ $(\dot{\Omega})$.

As in [5], we understand $W_2^\gamma(\Gamma_0)$ to be the complex Hilbert space of Sobolev—Slobodetskii with norm

$$\|\mathbf{u}\|_\gamma^2 = \sum_{|q|\leqslant|\gamma|} \|\mathbf{u}^{(q)}\|^2 + \sum_{|q|=[\gamma]} \int_{\Gamma_0}\int_{\Gamma_0} \frac{|\mathbf{u}^{(q)}(x_1) - \mathbf{u}^{(q)}(x_2)|^2}{|x_1 - x_2|^{2+2(\gamma+[\gamma])}} d\Gamma_{x_1} d\Gamma_{x_2}$$

$$\int_{\Gamma_0} \mathbf{u} d\Gamma = 0$$

where $[\gamma]$ is the integral part of γ, $W_2^{-\gamma}(\Gamma_0)$ is the space conjugate with W_2^γ (Γ_0) with respect to the scalar product in the space $W_2^\circ(\Gamma_0) = L_2(\Gamma_0) \ominus \{1\}$.

3. **The theorem of existence and uniqueness of solution.** It was shown in [7, 8] that the operator $(I + B)^{-1}$ is the self-conjugate positive definite inverse operator of operator $(I + B)$.

Below we shall use the following lemmas proved in [5].

Lemma 1. Operator $C = \Gamma T$ isothermally maps $W_2^{-1/2}(\Gamma_0)$ on $W_2^{1/2}(\Gamma_0)$, whose contraction on $W_2^\circ(\Gamma_0) = L_2(\Gamma_0) \ominus \{1\}$ is a self-conjugate positive entirely continuous operator acting in $W_2^\circ(\Gamma_0)$.

Lemma 2. Operator $B_2 = C^{1/2}B_1C^{1/2}$ is unboundedly self-conjugate and positive definite in $W_2^\circ(\Gamma_0)$, and $D(B_2) = W_2^1(\Gamma_0)$. Operator B_2^{-1} belongs to class σ_q for $q > 3$.

Lemma 1 implies that operator C has the inverse operator C^{-1} which is self-conjugate and positive definite in $W_2^\circ(\Gamma_0)$. A direct check shows that $W_2^{1/2}(\Gamma_0)$ is the determining region of operator $C^{-1/2}$.

Applying operator $(I + B)^{-1}$ to both sides of Eqs. (2.1) we obtain

$$\frac{d\mathbf{s}}{dt} + \nu^{-1}\sigma^{1/2}A^{1/2}TC^{-1/2}\frac{d\eta}{dt} + \nu A^{1/2}(I + B)^{-1}A^{1/2}\mathbf{s} = 0 \qquad (3.1)$$

$$\nu^{-1}\sigma^{1/2}\frac{d\eta}{dt} + \nu^{-1}\sigma C^{1/2}B_1\Gamma(A^{-1/2}\mathbf{s} + \nu^{-1}\sigma^{1/2}TC^{-1/2}\eta) = 0$$

$$\mathbf{s} = A^{1/2}\mathbf{v}, \quad \eta = \nu\sigma^{-1/2}C^{1/2}\varphi$$

We consider the obtained system as a single ordinary differential equation of the first order in $L_2^\circ(\Omega) \oplus W_2^{1/2}(\Gamma_0)$, acting on which with the operator

$$\begin{Vmatrix} I_\Omega & -A^{1/2}TC^{-1/2} \\ 0 & \nu\sigma^{-1/2}I_\Gamma \end{Vmatrix}$$

we obtain an equation of the form

$$d\mathbf{x}/dt + M_1\mathbf{x} + K_1\mathbf{x} = 0 \qquad (3.2)$$

$$M_1 = \begin{Vmatrix} \nu A^{1/2}(I + B)^{-1}A^{1/2} & 0 \\ 0 & \nu^{-1}\sigma B_2 \end{Vmatrix}, \quad \mathbf{x} = \begin{Vmatrix} \mathbf{s} \\ \eta \end{Vmatrix}$$

$$K_1 = \begin{Vmatrix} -\nu^{-1}\sigma A^{1/2}TB_1\Gamma A^{-1/2} & -\nu^{-2}\sigma^{3/2}A^{1/2}TC^{-1/2}B_2 \\ \sigma^{1/2}C^{1/2}B_1\Gamma A^{-1/2} & 0 \end{Vmatrix}$$

where I_Ω and I_Γ are unit operators in $L_2^\circ(\Omega)$ and $W_2^{1/2}(\Gamma_0)$, respectively.

The initial conditions become

$$x(0) = x_0 = \text{col}\,\{A^{1/2} v_0,\ \sigma^{-1/2}\nu C^{1/2}\varphi_0\}, \quad u_0 = v_0 + T\varphi_0 \quad (3,3)$$

Theorem 1. Equation (3.2) is an abstract parabolic equation in space $L_2^\circ(\Omega) \oplus W_2^{1/2}(\Gamma_0)$, whose Cauchy problem is uniformly correct. The related semigroup is analytic in the sector containing the positive axis. Problem (3.2), (3.3) has a weakened solution for any x_0.

Proof. Operator $A^{-1/2}(I + B)A^{-1/2}$ is bounded, self-conjugate, and positive, hence its inverse operator $A^{1/2}(I + B)^{-1}A^{1/2}$ is self-conjugate positive definite in $L_2^\circ(\Omega)$. The last of formulas (2.1) implies that the region of values of operator B is the same as the region of values of the projection operator Π on the set of linear functions of the form $r \times l$, hence according to [9] the region of values of B consists of functions that are as smooth as desired. It follows from this that the region of values of operator $A^{-1/2}(I + B)A^{-1/2}$ consists of functions $v \in W_2^2(\Omega)$, and the determining region of operator $A^{1/2}(I + B)^{-1} A^{1/2}$ is $W_2^2(\Omega) \cap W_2^{1,0}(\Omega)$.

Using operator B_2 we form the scale of Gilbert spaces [10, 11]

$$H_\gamma(\Gamma) = D(B_2^\gamma)$$

By Lemma 2 this scale applies to Gilbert spaces that join $W_2^\circ(\Gamma_0)$ and $W_2^1(\Gamma_0)$. From this and [11] follows that for $\gamma \leqslant 1$ the Sobolev spaces $W_2^\gamma(\Gamma_0)$ are the same as spaces $H_\gamma(\Gamma_0)$.

Since operator B_2 is self-conjugate and positive definite in $H_0(\Gamma_0) = W_2^\circ(\Gamma_0)$, hence according to [11] it is self-conjugate and positive definite also in $H_{1/2}(\Gamma_0) = W_2^{1/2}(\Gamma_0)$.

Thus operator M_1 is self-conjugate and positive definite in $L_2^\circ(\Omega) \oplus W_2^{1/2}(\Gamma_0)$. Hence the semigroup generated by the equation $dx/dt = -M_1 x$, is contractive and analytic in the left half-plane (see [12]).

Let us represent operator K_1 in the form

$$K_1 = T_1 M_1$$

$$T_1 = \begin{Vmatrix} -\sigma\nu^{-2}T_{11} & -\sigma^{1/2}\nu^{-1}T_{12} \\ \sigma^{1/2}\nu^{-1}T_{21} & 0 \end{Vmatrix}$$

$$T_{11} = T_{12}T_{21}, \quad T_{12} = A^{1/2}TC^{-1/2}, \quad T_{21} = B_2 C^{-1/2}\Gamma A^{-1}(I + B)A^{-1/2}$$

Operator K_1 is entirely subordinated to operator M_1, if operator T_1 is entirely continuous in the space $L_2^\circ(\Omega) \oplus W_2^{1/2}(\Gamma_0)$ (see, e.g., [12]). This statement follows from the following lemma.

Lemma 3. Operator T_{12} acts entirely continuously from $W_2^{1/2}(\Gamma_0)$ in $L_2^\circ(\Omega)$. To prove this let us assume that μ is a bounded set in $W_2^{1/2}(\Gamma_0)$. It follows from Lemma 1 that operator $C^{-1/2}$ transforms μ into a bounded set in $W_2^\circ(\Gamma_0)$ and into a compact set in $W_2^{-1/2}(\Gamma_0)$ by virtue of complete continuity of the imbedding operator. The above properties of operators T and A imply that operator $TC^{-1/2}$ transforms μ into a compact set in $W_2^1(\Omega)$, and operator $A^{1/2}TC^{-1/2}$ transforms μ into a compact set in $L_2^\circ(\Omega)$. This implies total continuity of operator $A^{1/2}TC^{-1/2}$ from $W_2^{1/2}(\Gamma_0)$ to $L_2^\circ(\Omega)$. The lemma is proved.

Lemma 4. Operator T_{11} acts entirely continuously from $L_2^\circ(\Omega)$ to $L_2^\circ(\Omega)$.

P r o o f. Since the region of values of operator B consists of functions that are as smooth as required operator $(I + B)\, A^{-1/2}$ is bounded as an operator from $L_2^\circ(\Omega)$ in $W_2^1(\Omega)$ and by the theorem on imbedding is entirely continuous as an operator from $L_2^\circ(\Omega)$ in $W_2^{1/2}(\Omega)$. Operator $B_2C^{-1/2}\Gamma A^{-1} = C^{1/2}B_1\Gamma A^{-1}$ is bounded as an operator from $W_2^{1/2}(\Omega)$ in $W_2^{1/2}(\Gamma_0)$, as implied by the mapping chain

$$W_2^{1/2}(\Omega) \xrightarrow{A^{-1}} W_2^{5/2}(\Omega) \xrightarrow{\Gamma} W_2^2(\Gamma_0) \xrightarrow{B_1} W_2^0(\Gamma_0) \xrightarrow{C^{1/2}} W_2^{1/2}(\Gamma_0)$$

Continuity of the operator in the first link was shown in [13], in the second from the theory of traces [13] and in the third from the estimates [14]

$$\|B_1^{-1}\mathbf{v}\|_{W_2^2(\Gamma_0)} \leqslant c\|\mathbf{v}\|_{W_2^0(\Gamma_0)} \quad \text{for} \quad \mathbf{v} \in W_2^{\prime 0}(\Gamma_0)$$

The operator in the fourth link is continuous by virtue of Lemma 1.

Thus operator T_{11} represents the product of the entirely continuous operator $(I + B)\, A^{-1/2}$ from $L_2^\circ(\Omega)$ in $W_2^{1/2}(\Omega)$, the bounded operator $B_2C^{-1/2}\Gamma A^{-1}$ from $W_2^{1/2}(\Omega)$ in $W_2^{1/2}(\Gamma_0)$ and of the entirely continuous operator T_{12} from $W_2^{1/2}(\Gamma_0)$ in $L_2^\circ(\Omega)$, hence it is entirely continuous as an operator from $L_2^\circ(\Omega)$ in $L_2^\circ(\Omega)$. The lemma is proved.

L e m m a 5. Operator T_{21} acts entirely continuously from $L_2^\circ(\Omega)$ in $W_2^{1/2}(\Gamma_0)$.

Proof of this lemma follows from the proof of Lemma 4.

The total subordination of operator K_1 to operator M_1 is proved. Then all assertions of Theorem 1 follow from [12].

R e m a r k. For a specified intitial distribution of fluid velocity $\mathbf{u}_0$ in region Ω and initial deviation of the free surface N_0 from the equilibrium surface (1.6), p_0 on surface Γ_0 at the initial instant of time is determined by the second of formulas (1.3). As shown in [6], the initial values $\mathbf{v}_0$ and $\mathbf{w}_0 = T\varphi_0$ are, consequently, determined by expansion (3.3) in $\mathbf{u}_0$ and p_0.

4. N o r m a l o s c i l l a t i o n s. Let us consider normal oscillations of a viscous fluid in the simultaneous motion of the system body + fluid under conditions of total weightlessness. We seek a solution of the problem of the form

$$(\mathbf{u},\ p,\ N) = e^{-\lambda t}(\mathbf{u}_1,\ p_1,\ N_1)$$

where $\mathbf{u}_1$, p_1, and N_1 are functions of coordinates only. For the quantities $\mathbf{s}_1 = A^{1/2}\mathbf{v}_1$ and $\eta_1 = \sigma^{-1/2}\nu C^{1/2}\varphi_1$ we obtain the problem

$$\nu\mathbf{s}_1 = \lambda F_1(\mathbf{s}_1 + \nu^{-1}\sigma^{1/2}T_{12}\eta_1) \tag{4.1}$$

$$\sigma^{1/2}\nu^{-1}T_{12}\eta_1 = \sigma(\lambda\nu)^{-1}F_2(\mathbf{s}_1 + \sigma^{1/2}\nu^{-1}T_{12}\eta_1)$$

$$F_1 = A^{-1/2}(I + B)\, A^{-1/2}, \quad F_2 = A^{1/2}TB_1\Gamma A^{-1/2}$$

Using the notation $\xi_1 = \mathbf{s}_1 + \sigma^{1/2}\nu^{-1}T_{12}\eta_1$ we have

$$\xi_1 = \lambda\nu^{-1}F_1\xi_1 + \sigma(\lambda\nu)^{-1}F_2\xi_1 \tag{4.2}$$

Multiplying both sides by ξ_1 and noting that owing to the positiveness of operator F_1 we have $(F_1\xi_1,\ \xi_1) \geqslant 0$ and owing to the positive definiteness of operator B_1, $(F_2\xi_1,\ \xi_1) = (B_1\Gamma A^{-1/2}\xi_1,\ \Gamma A^{1/2}\xi_1) > 0$, we obtain

$$\lambda={}^1/_2\{\nu(\xi_1,\ \xi_1)\pm[\nu^2(\xi_1,\ \xi_1)^2-4\sigma(F_1\xi_1,\ \xi_1)(F_2\xi_1,\ \xi_1)]^{1/2}\}/(F_1\xi_1,\ \xi_1)$$

which shows that λ has a positive real part.

To prove the completeness of the system of the eigen- and adjoint vectors of problem (4.1) we use Eq. (3.2) which for normal oscillations is of the form

$$x_1 = \lambda M_1^{-1}(I+S)x_1 \tag{4.3}$$

$$M_1^{-1} = \begin{Vmatrix} \nu^{-1}F_1 & 0 \\ 0 & \sigma^{-1}\nu B_2^{-1} \end{Vmatrix}, \quad x_1 = \begin{Vmatrix} s_1 \\ \eta_1 \end{Vmatrix}$$

$$S = \begin{Vmatrix} 0 & \nu^{-1}\sigma^{1/2}T_{12} \\ -\sigma^{1/2}\nu^{-1}T_{21} & -\sigma\nu^{-2}T_{21}T_{12} \end{Vmatrix}$$

Lemma 6. Operator S is entirely continuous in $L_2^\circ(\Omega) \oplus W_2^{1/2}(\Gamma_0)$.

Proof. By Lemmas 3 and 5 operator T_{12} is entirely continuous as an operator from $W_2^{1/2}(\Gamma_0)$ in $L_2^\circ(\Omega)$, and operator T_{21} is entirely continuous as an operator from $L_2^\circ(\Omega)$ in $W_2^{1/2}(\Gamma_0)$. It remains to show that the operator $T_{21}T_{12}$ is entirely continuous in $W_2^{1/2}(\Gamma_0)$. It follows from the proof of Lemmas 3 and 4 that it is the product of the entirely continuous operator $TC^{-1/2}$ from $W_2^{1/2}(\Gamma_0)$ in $W_2^{1/2}(\Omega)$ of the bounded operator $(I+B)$ in $W_2^{1/2}(\Omega)$, and of the bounded operator $B_2C^{-1/2}\Gamma A^{-1}$ from $W_2^{1/2}(\Omega)$ in $W_2^{1/2}(\Gamma_0)$. Hence it is entirely continuous as an operator from $W_2^{1/2}(\Gamma_0)$ in $W_2^{1/2}(\Gamma_0)$. The lemma is proved.

As shown above the self-conjugate operator A^{-1} is entirely continuous and belongs to class σ_q ($\forall\ q > {}^3/_2$). Operator F_1 has the same properties. By Lemma 2 and [11] operator B_2^{-1} is entirely continuous, self-conjugate in $W_2^{1/2}(\Gamma_0)$, and belongs to class σ_q ($\forall q > 3$). This implies that operator M_1^{-1} is self-conjugate, entirely continuous in $L_2^\circ(\Omega) \oplus W_2^{1/2}(\Gamma_0)$, and belongs to class σ_q with $q > 3$.

Theorem 2. The system of eigen- and adjoint vectors of problem (4.1) of viscous fluid normal oscillations in the simultaneous motion of the system body + fluid under conditions of total weightlessness is closed in $L_2^\circ(\Omega) \oplus W_2^{1/2}(\Gamma_0)$. All normal oscillations for any ε (>0) are damped, except possibly a finite number of those whose argument is comprized within the angle $-\varepsilon < \arg\lambda < \varepsilon$. The spectrum of this problem is discrete and has a bunching point at infinity.

Theorem 2 flows from Keldysh's theorem [15].

The author thanks S. G. Krein for valuable guidance and constant interest in this work and, also, N. D. Kopachevskii for useful remarks.

REFERENCES

1. Moiseev, N. N. and Rumiantsev, V. V., Dynamics of a Body with Cavities Containing Fluid. Moscow, "Nauka", 1965.

2. Chernous'ko, F. L., Motion of a Solid Body with Cavities Containing a Viscous Fluid. Moscow, VTs Akad. Nauk SSSR, 1968.

3. Krein, S. G. and Ngo Zui Kan, The problem of small motions of a body with a cavity partially filled with a viscous fluid. PMM, Vol. 33, No. 1, 1969.

4. K o p a c h e v s k i i, N. D., On the oscillations of a rotating capillary viscous fluid. Dokl. Akad. Nauk SSSR, Vol. 219, No. 5, 1974.

5. B a b s k i i, V. G., K o p a c h e v s k i i, N. D., M y s h k i s, A. D., S l o b o z h a n i n, L. A., and T i u p t s o v, A. D., Hydromechanics of Weightlessness. Moscow, "Nauka", 1976.

6. K r e i n, S. G. and L a p t e v, G. I., On the problem of motion of a viscous fluid in an open vessel. Funktsional'nyi Analiz i ego Prilozheniia, Vol. 8, No. 4, 1968.

7. N g o Z u i K a n, On the motion of a solid body with cavities filled with incompressible viscous fluid (English translation) J.Comput. Math. and Math. Phys., Pergamon Press, Vol. 8, No. 4, 1968.

8. K o b r i n, A. I., On the motion of a hollow body filled with viscous liquid about its center of mass in a potential body-force field. PMM, Vol. 33, No. 3, 1969.

9. B y k h o v s k i i, E. B. and S m i r n o v, N. V., On the orthogonal expansion of vector function space quadratically summable over a specified region, and on operators of vector analysis. Tr. of the Steklov Matem. Inst. Akad. Nauk. SSSR, Vol. 59, 1960.

10. Functional Analysis (Ed. S. G. Krein). Moscow, "Nauka", 1972.

11. K r e i n, S. G. and P e t u n i n, Iu. I., Scales of Banach spaces. Uspekhi Matem. Nauk., Vol. 21, No. 2, 1966.

12. K r e i n, S. G., Linear Differential Equations in the Banach space. Moscow, "Nauka", 1967.

13. V o l e v i c h, L. R., Solvability of boundary value problems for general elliptic systems. Matem. Sb., Vol. 68, No. 3, 1965.

14. A g r a n o v i c h, M. S., Elliptic singular integro-differential operators. Uspekhi Matem. Nauk, Vol. 20, No. 5, 1965.

15. G o k h b e r g, I. Ts. and K r e i n, S. G., Introduction to the Theory of Linear nonself-conjugate operators in Hilbert space, Moscow, "Nauka", 1965. (see also English translation, Providence, American Math. Society, 1970).

Translated by J. J. D.